TRANSVERSE DIFFUSION AND HEAT CONDUCTION IN A GRANULAR LAYER

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The process of non-steady-state transverse diffusion of a passive additive in a granular layer described by a cellular model is investigated. The general results obtained in [1] are applied to an analysis of concrete transport processes of matter and heat in a granular layer. The following four cell models are treated: (1) ideal mixing cells without stagnation zones; (2) ideal mixing cells with stagnation zones; (3) ideal mixing cells with diffusive stagnation zones; (4) ideal mixing cells with diffusive stagnation zones having a finite exchange rate between the free volume and the stagnation zone. The conditions of applicability for each of the above models are found. The time to establish a normal distribution in the transverse diffusion process is determined for all the models. This quantity is then connected with the physical characteristics of transport processes of matter in a layer of non-porous and porous particles, the transport of heat in a granular layer, and the transport of matter in a layer of particles which adsorb an additive.

1. GENERAL RESULTS

The process of transverse diffusion of a passive additive in a granular layer was investigated in [1]. In accordance with the model adopted there the layer is treated as an aggregate of cells joined by channels. The fluid or gas crossing the layer flows out of the cells of each horizontal level into the cells of the following level in the direction of the flow, each time experiencing a random transverse displacement by some distance which is determined by the packing structure of the layer. Each cell is characterized by some differential distribution function of time spent in the cell $f(t)$, which in what follows will be called a micro-distribution. The form of the microdistribution, which is determined by the physical processes inside the cell, was not given a specific form in [1]. It was found that the Laplace form for the probability of displacement of additive particles by m cells in a direction normal to the direction of the stream has the form

$$F_m(p) = \frac{g^{-1} - 1}{p\sqrt{g^{-2} - 1}} \left(g^{-1} - \sqrt{g^{-2} - 1}\right)^m$$

$$\left(g(p) = \int_0^\infty f(t)\, e^{-pt}\, dt\right). \tag{1.1}$$

Here g(p) is the characteristic function of the micro-distribution. The function $F_m(t)$ in what follows is called the macrodistribution. The basic characteristics of the macrodistribution are its dispersion $\mu_2(t)$ and kurtosis coefficient Ex(t), which characterizes the departure of the distribution from a normal law. In [1] the following asymptotic formulas for these two quantities were found:

$$\mu_2(t) = \frac{t}{\alpha_1} + \frac{\alpha_2}{2\alpha_1^2} \qquad \left(t \gg t_1 = \frac{-1}{\operatorname{Re} p_1}\right),$$

$$Ex(t) = \frac{\mu_4(t)}{\mu_2^2(t)} - 3 =$$

$$= \left(\frac{t}{\alpha_1} + \frac{\alpha_2}{2\alpha_1^2}\right)^{-2} \left[\frac{t}{\alpha_1}\left(1 + \frac{3\alpha_2}{\alpha_1^2}\right) + \frac{\alpha_2}{\alpha_1^3} + \frac{15}{4}\frac{\alpha_2^2}{\alpha_1^4} - \frac{2\alpha_3}{\alpha_1^3}\right]. \tag{1.2}$$

Here α are the coefficients in the series expansion of the Taylor function

$$\Delta(p) = g^{-1} - 1 = \sum_{j=1}^{\infty} \alpha_j (-1)^{j+1} \frac{p^j}{j!}. \tag{1.3}$$

Formulas (1.2) are valid for times $t \gg t_1$, while p_1 ($\operatorname{Re} p_1 < 0$) is the zero of the function $\Delta(p)$ nearest to the imaginary axis. It follows from the asymptotic formulas that as $t \to \infty$ a normal distribution is established with dispersion t/s, where s is the average time spent in a cell, regardless of the form of the microdistribution. However, we can say nothing about the speed of establishing a normal distribution without specifying the form of the function $f(t)$ or $\Delta(p)$. In what follows, a series of concrete models is treated, leading to specific forms of microdistribution functions, and at the same time the connection is established between the parameters which appear in the general formulas (1.2) and the physical characteristics of the granular layer and the stream which flows through it.

2. INVESTIGATION OF THE MODELS

2.1°. Ideal mixing cells (model 1). The model with cells of an ideal solution [2] is the simplest. For this model the characteristic function of the microdistribution and the macrodistribution function are specified by the formulas

$$g(p) = (1 + ps)^{-1}, \qquad F_m(t) = I_m(t/s) e^{-t/s}. \tag{2.1}$$

Here I_m is a modified Bessel function of the first kind of order m. Formulas for the moments of the microdistribution and the kurtosis coefficient may be obtained directly from (2.1) as well as from expressions (1.2). In the case under consideration these are not asymptotic but exact formulas, since the characteristic function (2.1) is nowhere equal to unity except at the point p = 0. We have

$$\mu_2(t) = t/s, \qquad \mu_4(t) = t/s + 3t^2/s^2,$$

$$\operatorname{Ex}(t) = s/t. \tag{2.2}$$

It is clear from (2.2) that a normal distribution is established for times $t \gg s$. We notice that during a time t the front of the stream manages to pass through n = t/s cells along the layer. In the case being considered a normal distribution in the transverse diffusion process is established after $n \gg 1$ cells have been traversed. This is the maximum possible speed for approaching a normal law, since for times $t \sim s$ and lengths of the order of the cell dimension it is, in general, meaningless to talk about a macrodistribution law.

2.2°. Ideal mixing cells with stagnation zones (model 2). We shall consider a cell composed of two regions, a free volume and a stagnation zone. The flow of gas or fluid passes through the free volume of the cell only. For Reynolds numbers which are not too small ($R = ul/\nu \geqslant 50$, where u is the linear velocity of the stream, l is the characteristic dimension of the cell or the diameter of a granule, and ν is the kinematic viscosity) the free volume of the cell may be taken to be ideally mixed. The concentration of additive (marker) c_1, which is constant throughout the entire free volume, and its variation with time is determined by

$$dc_1 / dt = c_1 / t_0 - I\sigma . \qquad (2.3)$$

Here t_0 is the average time spent in the free volume, equal to the ratio of the free volume to the volume rate of flow, I is the flow from the free volume into the stagnation zone through their border, which is of area σ per unit free volume. In the model considered here it is assumed that the concentration of marker in the stagnation zone c_2 is independent of the spatial coordinates and is a function of time only.

The model of stagnation zones has been proposed previously in connection with the treatment of the process of longitudinal diffusion [3]. The equations and initial conditions for the concentrations c_1, and c_2 have the form

$$\frac{dc_1}{dt} = -\frac{c_1}{t_0} - \alpha q (c_1 - c_2), \qquad \frac{dc_2}{dt} = q (c_1 - c_2),$$

$$c_1(0) = \frac{1}{V_1}, \quad c_2(0) = 0 . \qquad (2.4)$$

Here q is the exchange rate constant between the stagnation zone and the free volume, relative to the volume of the stagnation zone V_2, α is the ratio of the stagnation zone volume V_2 to the free volume of the cell V_1. The system of equations (2.4) may be solved to find the distribution function $f(t)$ for times spent by a particle in a cell, which coincides with the function $c_1(t)$ except for the normalizing factor V_1/t_0. Calculating the characteristic micro-distribution function for the given model we determine Δ (p)

$$\Delta\ (p) = pt_0\,(p+q)^{-1}\ \ [p + q\ \ (1\ \alpha)]$$

$$\Delta\ (p) = 0 \quad \text{for} \ \ p_0 = 0 \text{ and } p_1 = q\,(1+\alpha) . \qquad (2.5)$$

Using (2.5) we have

$$\alpha_1 = s = t_0\,(1+\alpha),$$

$$\alpha_2 = \frac{2\alpha}{1+\alpha}\frac{s}{q}, \quad \alpha_3 = \frac{6\alpha}{1+\alpha}\frac{s}{q^2} . \qquad (2.6)$$

Since the quantities α_1, α_2, and α_3 are known from formulas (1.2) we can find asymptotic expressions for the dispersion and kurtosis coefficient of the macro-distribution for the given granular layer model, valid for times $t \gg t_1$:

$$\mu_2\,(t) = \frac{t}{s} + \frac{\alpha}{(1+\alpha)\,qs} \qquad \left(t \gg t_1 = \frac{1}{q\,(1+\alpha)}\right),$$

$$Ex\,(t) = \left(\frac{t}{s} + \frac{\alpha}{1+\alpha}\frac{1}{qs}\right)^{-2}\left[\frac{t}{s}\left(1 + \frac{6\alpha}{1+\alpha}\frac{1}{qs}\right) + \right.$$

$$\left. + \frac{2\alpha}{(1+\alpha)\,qs} + \frac{15\alpha^2}{(1+\alpha)^2(qs)^2} - \frac{12\alpha}{(1+\alpha)\,(qs)^2}\right] . \qquad (2.7)$$

On examining expressions (2.7) we see that the kurtosis coefficient Ex, which characterizes the departure of the distribution from the normal law becomes small compared with unity after a time $t \gg t_s = \max[s, q^{-1}]$. It may easily be seen that when the condition $Ex(t) \ll 1$ is fulfilled it is only necessary to retain the first term in the formula for $\mu_2(t)$. In order to explain the meaning of the results obtained we shall write down the exact formula for the dispersion of the macrodistribution of the model in question and examine its variation in time

$$\mu_2\,(t) = \frac{t}{s} + \frac{\alpha}{(1+\alpha)\,qs}\,[1 - e^{-q(1+\alpha)t}] . \qquad (2.8)$$

In the beginning for $t \ll [q(1+\alpha)]^{-1}$ the dispersion increases with time as t/t_0. This period corresponds physically to the time when the additive has not yet managed to penetrate into the stagnation zone. The increase in the dispersion subsequently slows down; for $t \gg [q(1+\alpha)]^{-1}$ the exponent in (2.8) vanishes and the asymptotic formula (2.7) results. The nature of further variations of dispersion with time depends strongly on the size of the parameter α. If $\alpha \leqslant 1$, then $\mu_2(t) = t/s \sim t/t_0$, i.e., the change in the dispersion follows practically the same law as for times which are small. The picture for $\alpha \gg 1$ is quite different. In this case during an interval of time $[q(1+\alpha)]^{-1} \ll t \ll q^{-1}$ the dispersion remains constant and equal to $(qs)^{-1}$, after which for $t \gg q^{-1}$ it begins to increase very slowly, compared with the initial increase, according to the law t/s. The results given above are illustrated graphically in a figure in which the dispersion of the macro-distribution is given as a function of time.

For $\alpha \gg 1$ during a period $t_s \gg t \gg t_1$ corresponding to a constant dispersion the additive, on falling into stagnation zones of very large volume, experiences practically no transverse displacement. After the passage of a time t_s a dynamical equilibrium is established between the cells and the stagnation zones and they operate together like a single cell of very large capacity.

2.3°. Ideal mixing cells with diffusion stagnation zones (model 3). We shall now consider stagnation zones in which the transport velocity of the additive can no longer be taken to be infinite. In this case the concentration of marker within a stagnation zone will depend not only on time but also on the spatial coordinates, and will satisfy the equation of molecular diffusion. We shall take the stagnation zone to be a flat layer of thickness δ, and let the x-axis lie so that $x = 0$ is on the boundary between the stagnation zone and the free volume, and $x = \delta$ is on the "sealed" boundary. Then the equation for c_2 and its initial and boundary conditions assume the form

$$D\frac{\partial^2 c_2}{\partial x^2} = \frac{\partial c_2}{\partial t}, \qquad D\frac{\partial c_2}{\partial x}\Big|_{x=\delta} = 0, \qquad \begin{matrix} c_2\,(0,\,t) = c_1\,(0,\,t) \\ c_2\,(x,\,0) = 0 . \end{matrix} \quad (2.9)$$

On solving Eq. (2.9) together with (2.3) we find g(p) and Δ (p) corresponding to the given model

$$\Delta\,(p) = pt_0 + \varepsilon\sigma t_0\sqrt{Dp}\,\operatorname{th}\sqrt{\frac{p}{D}}\,\delta = pt_0\left(1 + \alpha\frac{\operatorname{tg}\lambda}{\lambda}\right)$$

$$\left(\alpha = \varepsilon\sigma\delta,\ \lambda = \delta\sqrt{\frac{p}{D}}\right) . \qquad (2.10)$$

Here ε is the ratio of the specific volumes of the stagnation zone and the free volume. The characteristic time t_1 which must elapse before the asymptotic formulas (1.2) can be employed is determined from the solution of the transcendental equation

$$\alpha \operatorname{tg} \lambda + \lambda = 0, \qquad t_1 = \frac{1}{\lambda_1^2} \frac{\delta^2}{D} = \frac{t_d}{\lambda_1^2}$$

$$({}^1/{}_2\pi < \lambda_1 < \pi). \tag{2.11}$$

For $\Delta(p)$ determined from formula (2.10) we have

$$\alpha_1 = s = t_0(1 + \alpha), \qquad \alpha_2 = \frac{2}{3} \frac{\alpha}{1 + \alpha} st_d,$$

$$\alpha_3 = \frac{4}{5} \frac{\alpha}{1 + \alpha} st_d^2. \tag{2.12}$$

Formulas (2.12) coincide formally with formulas (2.6), apart from numerical coefficients, if we take t_d equal to q^{-1}. Consequently we may immediately conclude that $t_s = \max [s, t_d]$, and also that when the condition $t \gg t_s$ is fulfilled the dispersion is equal to t/s. It should be stressed that in spite of the fact that formulas (2.6) and (2.12) are formally the same, the models which lead to these formulas are fundamentally different. This difference is particularly apparent for $\alpha \gg 1$. In the latter case, model 3 cannot lead to a function of time for which there exists a time interval with a constant dispersion.

2.4°. Ideal mixing cells with diffusive stagnation zones having a finite exchange rate between the free volume and the stagnation zone (model 4). When there is a finite rate of exchange between the free volume of the cell and the stagnation zone the boundary condition must be changed to

$$\frac{I}{\varepsilon} = D \frac{\partial c_2}{\partial x} = k(c_2 - c_1) \qquad \text{for } x = 0. \tag{2.13}$$

Here k is the exchange coefficient. Allowing for (2.13) we have

$$\Delta(p) = pt_0 + \alpha B \frac{t_0}{t_d} \left(B \frac{\operatorname{cth} \sqrt{pt_d}}{\sqrt{pt_d}} + 1 \right)^{-1}. \tag{2.14}$$

(The Biot number $B = k\delta/D$.)

It is not difficult to see that (2.14) contains formulas (2.5) and (2.10) as limiting cases. In order to obtain (2.10) it is necessary to pass to the limit $k \to \infty$ in (2.14). Formula (2.5) corresponding to stagnation zones of ideal mixing is obtained from the general formula in the limit when $D \to \infty$, while in this case the parameter q of model 2 turns out to be equal to k/δ.

The zeros of the function $\Delta(p)$ are found by solving the transcendental equation

$$\lambda^2 - B\lambda \operatorname{ctg} \lambda - \alpha B = 0, \tag{2.15}$$

which is a generalization of Eq. (2.11) of model 3. The characteristic time t_1 is connected with the root λ_1 of Eq. (2.15) having the smallest modulus, by the relation $t_1 = t_d/\lambda_1^2$, similar to that which was obtained for model 3. However, as distinct from the latter case, λ_1 now depends on the value of the parameter B and in the present case it is impossible to draw the conclusion that $\lambda_1 \sim 1$ for all values of B.

When Eq. (2.15) is examined we see that the root λ_1 can be small compared with unity only when the following conditions are fulfilled simultaneously:

$$B \ll 1, \qquad \alpha B \ll 1. \tag{2.16}$$

If conditions (2.16) are fulfilled then the time t_1 (much greater than t_d) is determined by the same expression as was obtained for model 2. Thus fulfilment of inequalities (2.16) is an indispensable condition for the model of ideal mixing cells with stagnation zones to be applicable. In the case in which at least one of inequalities (2.16) is not fulfilled, the root λ_1 will be of the order unity, and the time $t_1 \sim t_d$. Using formula (2.14) we have

$$\alpha_1 = s = t_0(1 + \alpha), \ \alpha_2 = \frac{\alpha}{1 + \alpha} st_d \frac{2(B + 3)}{3B},$$

$$\alpha_3 = \frac{6\alpha}{1 + \alpha} st_d^2 \left[\frac{1}{45} + \left(\frac{B + 3}{3B} \right)^2 \right]. \tag{2.17}$$

It is not difficult to see that for $B \gg 1$ expressions (2.17) pass to those of (2.12), and for $B \ll 1$ to those of (2.6). From this it follows in particular that if the model of ideal mixing cells with stagnation zones is to be used then conditions (2.16) are not only necessary but also sufficient. The condition of applicability for model 3 is the inequality $B \gg 1$. If $B \sim 1$, then we must use the general formulas (2.17). Clearly $B = qt_d$, and so in the general case of arbitrary values of B the time for establishing a normal distribution is determined by the largest of the quantities s, t_d, q^{-1}.

3. PHYSICAL INTERPRETATION OF THE RESULTS

The outline which has been given is exceedingly general and can be applied to the analysis of transport processes of both matter and heat. In what follows we shall consider a series of specific processes.

3.1°. The transport of matter in a layer of nonporous particles. Here the regions close to the surface of the solid particles are stagnation zones where turbulent pulsations are damped and the transport of matter comes about only by means of molecular diffusion. A flat diffusive boundary layer at a hard surface [4] (of thickness δ, much less than the grain parameter l) constitutes such a stagnation zone. For a diffusion layer the quantity σ is the same as the specific surface of the hard particles and in order of magnitude is equal to l^{-1}. The quantity $\varepsilon = 1$, so that $\alpha \sim \delta/l \ll 1$. There is no resistance at the boundary of the free volume and the stagnation zone; thus $k = \infty$ and $B = \infty$. From this it follows immediately that the given process is described by model 3. In accordance with the results of ¶ 2.3°, the time for establishing a normal distribution is determined by the largest of the quantities s and t_d. Let us compare these quantities:

$$t_d/s = \delta^2/Dt_0 \sim RP/N^2 \sim R^{-0.2}P^{1/3}, \tag{3.1}$$

where $N = \delta/l$ is the Nusselt diffusion number and $P = \nu/D$ is the Prandtl diffusion number. In the estimate which has just been made the empirical function N(R, P) for a granular layer [5] has been used; we shall use this function in the following estimates also. It follows from (3.1) that the ratio $t_d/s \sim 1$ for gas fluxes and $t_d/s \sim 10$ for liquid fluxes; there appears to be practically no dependence on the Reynolds number. Clearly the ratio t_d/s determines the number of cells n_s which the stream front traverses before a normal distribution is established in the process of transverse diffusion. We note that in gases a normal distribution is established as rapidly as if stagnation zones were not present (model 1), and in liquids somewhat more slowly.

Another stagnation zone in a layer of nonporous particles is the region close to the point where the granules make contact [6]; this region is far from being flat. In a layer of spherical granules its border with the free volume of a cell is the side area of a cylinder of height δ_0, equal to the thicknes of the viscous sublayer, and of radius $(\delta_0 l)^{1/2}$ so that $\sigma \sim \delta_0^{3/2} l^{-5/2}$. The number of stagnation zones of this type which occur for a single cell is determined by the packing structure of the layer. Diffusion into the interior of such a stagnation zone is very difficult, and as was shown in paper [6] which dealt with longitudinal mixing in a granular layer, the influence of these zones on the erosion of the marker of neutral additive is somewhat stronger than for the stagnation zones close to the surface of the particles. The characteristic function for cells with stagnation zones close to the points where the grains make contact was obtained in paper [6]:

$$g(p) = \left\{1 + pt_0 + \varepsilon\sigma t_0 \sqrt{Dp} \left[\frac{I_0(\sqrt{pt_d})}{I_1(\sqrt{pt_d})} - \frac{2}{\sqrt{pt_d}}\right]\right\}^{-1}. \quad (3.2)$$

It may easily be shown that apart from numerical coefficients, results for this model may be obtained with the function (2.14) corresponding to model 3, if we set

$$\delta = \sqrt{\delta_0 l}, \quad \alpha = (\delta_0 / l)^2.$$

In this case

$$n_s = t_d / s \sim l\delta_0 / Dt_0 \sim RP^{4/3} / N \sim R^{0.4} P. \quad (3.3)$$

Here the difference between a liquid and a gas appears much more markedly than in (3.1). It follows from (3.3) that if the stagnation zones close to the points of contact do not play a notable part for gases, then for liquid streams a normal distribution is approached much more slowly and it is established only after the front of the stream has traversed a large number of cells $n \gg n_s \gg 1$. In this case if the number of cells along the layer is not very large ($n \leqslant n_s$), a normal distribution may not be established at all during the time that the front of the stream traverses the layer.

3.2°. **The transport of matter in a layer of porous particles.** Here the porous granule itself is a stagnation zone; the effect of the stagnation zones treated previously is negligibly small compared with the zone considered here. Since diffusion in a porous granule proceeds comparatively slowly, matter does not manage to penetrate deeply into the granule, and the geometry of the stagnation zone may be taken to be plane as before. It is now necessary to interpret D as the effective diffusion coefficient in a porous particle, and δ may be taken to be equal to its hydraulic radius. The exchange coefficient k is equal to the coefficient of mass transfer to the external surface of the granule. Straightforward estimates show [7] that in this case $B \gg 1$, so that the system under consideration may be described by model 3. The ratio of the specific volumes of the stagnation zone and the free volume of a porous particle is on the order of unity; on determining the parameter α it follows that for the given process $\alpha \sim 1$. An estimate of the number n_s shows that

$$n_s = t_d / s = \delta^2 / Ds \sim ul/D \sim RP. \quad (3.4)$$

It is clear from (3.4) that the effect of the stagnation zones is considerable even for gaseous fluxes.

3.3°. **Heat transfer in a granular layer.** In heat transfer solid particles also behave as stagnation zones. As in the previous model δ must be taken to be the hydraulic radius of a granule; D is the coefficient of thermal diffusivity of a solid particle. In the present case the coefficient ε is equal to the ratio of the heat capacities per unit volume of the hard sphere and the flux γ_p/γ_s. For gaseous fluxes $\varepsilon \gg 1$ and consequently $\alpha \gg 1$; in liquids normally $\varepsilon \sim 1$ and $\alpha \sim 1$. The exchange coefficient is equal to $k = k_t/\gamma_p$, where k_t is the coefficient of heat transfer to the outer surface of the granule. In this case the parameter B is equal to

$$B = k_t \delta / \chi_p \sim N\chi_s / \chi_p \sim R^{0.6} P^{1/3} \chi_s / \chi_p. \quad (3.5)$$

Here N is the thermal Nusselt number, P is the thermal Prandtl number, and χ_p and χ_s are the thermal conductivities of the solid particles and the stream, respectively. It follows from (3.5) that B may assume very varied values; however, small values of B are more characteristic for gaseous fluxes, and large values of B for liquid fluxes. Depending on the magnitude of the parameter B the system is described by one of the three models (2–4).

It should be noted that even though the thermal conductivity of the hard particles is comparatively large these act as before like stagnation zones. This comes about as the result of the fact that the heat conduction in the solid phase through the points of contact between the particles is very much inhibited [8]. Heat conduction from cell to cell through a solid particle also does not play a significant role, since

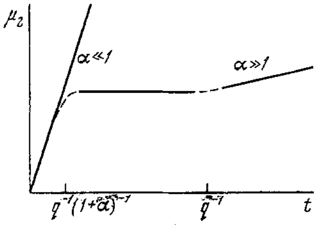
$$\varepsilon k/u \sim N/RP \sim R^{-0.4} P^{-2/3} \ll 1. \quad (3.6)$$

In accordance with the general results obtained for model 4, the time for establishing a normal distribution is determined by the largest of the quantities s, t_d, q^{-1}. Correspondingly n_s is determined by the largest of the quantities

$$1, \quad \frac{t_d}{s} \sim \frac{RP}{1+\alpha}, \quad \frac{1}{qs} \sim \frac{R^{0.4} P^{2/3}}{1+\alpha} \frac{\chi_p}{\chi_s} \sim \frac{t_d}{Bs}.$$

In gas fluxes for moderate Reynolds numbers ($R \sim 10^2 - 10^3$) ($t_d/s \leqslant 1$), and n_s may be considerably in excess of unity only if B is small enough, i.e., in a layer of particles having a high thermal conductivity. In this case the curve for the dispersion of the macrodistribution as a function of time may have a horizontal part (see figure). In liquids, $t_d/s \gg 1$, and so $n_s \gg 1$ always. It has already been noted that for liquids, small values of the Biot number are not characteristic, so that the quantity t_d/s is usually much larger than all the others, and $n_s = t_d/s$.

3.4°. Transfer of matter in a layer of nonporous particles which adsorb the additive. At a certain stage a surface of granules which adsorb the additive is a "stagnation zone." If the relation between the concentration of matter at the surface and the surface concentration of adsorbed material is determined by Henry's law with a constant a (having the dimensions of length), then the process under consideration is described by model 2, and we must set $q = k/a$, $\alpha = \sigma a$ in the corresponding formulas.



Here k must be taken to be the coefficient of mass transfer to the external surface of the granule. In accordance with the results of section 2.2°, the time for establishing a normal distribution is determined by the larger of the quantities s and a/k. The number of cells n_S in this case is equal to

$$n_s = \frac{a}{ks} \sim \frac{u}{k}\frac{a}{a+l} \sim \frac{a}{a+l} R^{0.4} P^{2/3}. \qquad (3.7)$$

It follows from formula (3.7) that the maximum possible value of $n_S = u/k$ is obtained for $a \gg 1$. However, we must not forget that together with the adsorption mechanism of the trapping of material within a cell, material is also trapped within a diffusion layer (see §3.1°), which leads to a value $n_S \sim u\delta/kl$. Comparing this quantity with (3.7), we see that the adsorption trapping mechanism is more effective than the diffusion mechanism for $a > \delta$. In the case of fairly strong adsorption ($a \gg l$) the macrodistribution dispersion function will have a plateau (see figure) for times $l/k \ll t \ll a/k$.

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